Gauge Invariance of Vector Mesons *

L. CASTELL **

Department of Applied Mathematics, University of Kent, Canterbury, England, and Trinity College, Dublin, Ireland

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It has been known for a very long time [1] that the MAXWELL equations for the free electromagnetic field are invariant not only under the inhomogeneous LORENTZ transformations, but in addition also under the dilatations and special conformal transformations (conformal group). This is a general property of every field equation for particles with arbitrary spin and mass zero [2]. Another property which in the conventional theory also depends on the mass being zero is the gauge invariance of the equations for the electromagnetic potentials [3]. So the question arises whether gauge invariance can be explained by conformal invariance.

The LIE algebra of the conformal group of MINKOWSKI space, and the algebra of SO(4,2) are isomorphic. The group SO(4,2) is the group of motions of a 5-dimensional pseudo-hyperbolic geometry. The suggestion that this 5-dimensional space of constant curvature can be considered as the generalized space-time continuum at subatomic level has been worked out in detail [4, 5]. The unitary representations [6, 7] of SO(4,2), which are carried by the solutions of the invariant field equations for tensors, in general give rise to a continuous mass spectrum $0 < m < \infty$.

The RIEMANNian metric $\bar{g}_{\mu\nu}$ of the 5-dimensional space is given by [4]

$$\bar{g}_{11}=\bar{g}_{22}=\bar{g}_{33}=-\bar{g}_{44}=\bar{g}_{55}=L^2/(y^5)^2,$$
$$\bar{g}_{\mu\nu}=0 \quad \text{for} \quad \mu \neq \nu.$$

(The fundamental length constant L is connected with the curvature $-1/L^2$. The range of the coordinates is given by

$$-\infty < y^j < +\infty, \quad 0 < y^5 < +\infty,$$

j, k etc. $= 1, 2, 3. 4.$)

The scalar wave equation

$$(\bar{g}^{\mu\nu} \nabla_\mu \nabla_\nu - C/L^2)\, \varphi = 0,$$

where ∇_μ abbreviates the covariant derivative, has already been discussed [4]. The equations for the vector potential A_ϱ can be written in the form

$$(\bar{g}^{\mu\nu} \nabla_\mu \nabla_\nu - C/L^2)\, A_\varrho = 0, \tag{1}$$

$$\bar{g}^{\mu\varrho} \nabla_\mu A_\varrho = 0, \tag{2}$$

which is equivalent to

$$\begin{aligned} &\left(\Box + \partial_5 \partial_5 - \frac{1}{y^5}\partial_5 - \frac{C+4}{(y^5)^2}\right) A_j = (2/y^5)\, \partial_j A_5, \\ &\left(\Box + \partial_5 \partial_5 - \frac{1}{y^5}\partial_5 - \frac{C+7}{(y^5)^2}\right) A_5 = (-2/y^5)\, g^{jk}\, \partial_j A_k, \\ &g^{jk}\, \partial_j A_k + \left(\partial_5 - \frac{3}{y^5}\right) A_5 = 0, \end{aligned} \tag{3}$$

$$\partial_\mu = \partial/\partial y^\mu, \quad \Box = g^{jk}\, \partial_j \partial_k,$$
$$g^{11}=g^{22}=g^{33}=-g^{44}=1,$$
$$g^{jk}=0 \quad \text{for} \quad j \neq k.$$

The plane wave solutions [8] of (3) are given by

$$A_j = e^{i(py)}\, y^5 \left(a_{j\perp} + \frac{i\, p_j}{m^2}\, a_5 (1 - y^5 \partial_5)\right) J_\nu(m\, y^5),$$
$$A_5 = e^{i(py)}\, (y^5)^2\, a_5\, J_\nu(m\, y^5),$$

where

$$(p\, y) = p_k\, y^k,$$
$$g^{jk}\, p_j\, p_k = -m^2,$$
$$(p\, a_\perp) = g^{jk}\, p_j\, a_{k\perp} = 0,$$
$$\nu = +\sqrt{C+5},$$

and $J_\nu(x)$ is the BESSEL function of order ν; from the one-valuedness of the unitary representation follows $\nu = 0, 1, 2, \ldots$. The field equation (1) and the generalized LORENTZ condition (2) therefore describe a LORENTZ covariant vectormeson (amplitude $a_{j\perp}$) which is coupled to an LORENTZ invariant scalar field (amplitude a_5).

Now for a real field A_μ the question arises whether (1) and (2) are invariant under the generalized gauge transformation [9]

$$A_\varrho \to A_\varrho + \nabla_\varrho f. \tag{4}$$

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** Present address: International Centre for Theoretical Physics, Trieste, Italy.

1 E. CUNNINGHAM, Proc. London Math. Soc. **8**, 77 [1910]. — H. BATEMAN, Proc. London Math. Soc. **8**, 223 [1910].

2 P. A. M. DIRAC, Ann. Math. **37**, 429 [1936]. — J. A. Mc LENNAN, Nuovo Cimento **3**, 1360 [1956].

3 As we are dealing with free field equations "mass" means always "bare mass". There are however gauge invariant non-local theories, where the bare photon mass is different from zero. See M. PRION, Nucl. Phys. **81**, 641 [1966], and literature quoted there.

4 L. CASTELL, Nuovo Cimento **46**, 1 [1966].

5 L. CASTELL, The Relativistic Position Operator at Subatomic Level, to be published.

6 Y. MURAI, Progr. Theor. Phys. **9**, 147 [1953]. — A. ESTEVE and P. G. SONA, Nuovo Cimento **32**, 473 [1964].

7 R. L. INGRAHAM, Nuovo Cimento **43**, 368, 483 [1966].

8 We can choose either the positive or the negative energy solution.

9 An alternative system of equations is given by

$$\bar{g}^{\mu\nu}\, \nabla_\mu F_{\nu\varrho} - [(C+4)/L^2]\, A_\varrho = 0, \tag{1'}$$

where $F_{\nu\varrho} = \nabla_\nu A_\varrho - \nabla_\varrho A_\nu$. Eq. (1') differs from (1) by

$$\nabla_\varrho (\bar{g}^{\mu\nu} \nabla_\mu A_\nu).$$

For $C \neq -4$ condition (2) follows from (1'), and both systems are equivalent. For $C = -4$ Eq. (1') is more general than (1) and (2): Eq. (1') is invariant under an arbitrary gauge transformation, and we are also free to choose a non-covariant gauge condition.

The real function f has to obey the two equations

$$\bar{g}^{\mu\varrho} \nabla_\mu \nabla_\varrho f = 0\,, \qquad (5)$$

$$(\bar{g}^{\mu\nu} \nabla_\mu \nabla_\nu - C/L^2) \nabla_\varrho f = 0\,. \qquad (6)$$

Both equations are compatible only if $C = -4$. Eq. (5) may be written in the form

$$\left(\Box + \partial_5 \partial_5 - \frac{3}{y^5} \partial_5 \right) f = 0\,.$$

So f is a superposition of solutions of the form

$$f^{(1)} = (y^5)^2 \, e^{i(py)} \, J_2(m\, y^5) + \text{h. c.}, \; (p\,p) = -m^2, \qquad (7)$$

$$f^{(2)} = e^{i(py)} + \text{h. c.}, \; (p\,p) = 0\,. \qquad (8)$$

With the help of the transformation generated by (7) we can always choose a gauge in which the component A_5 is zero, and we are left with a LORENTZ covariant vector meson. The generalized gauge transformation (4) does not necessarily depend on the condition mass $m=0$. The transformations generated by (8) will be of physical importance for the electromagnetic field in the conformal space. It is quite a surprising result however that they exist also for vector mesons.

We now make the step from the 5-dimensional space to the 4-dimensional conformal space by taking the limit [4] $y^5 \to 0$. First we look for formal plane wave solutions of the field Eqs. (3) with mass $m=0$. For $A_5 \neq 0$ there exist only solutions if $C=-4$, i. e. if our equations are gauge invariant.

$$A_j = e^{i(py)} \left(a_{j\perp} \, (y^5)^2 + b_{j\perp} + \frac{i\,p_j}{4} a_5 (y^5)^4 \right) + \text{h. c.},$$

$$A_5 = a_5 (y^5)^3 \, e^{i(py)} + \text{h. c.},$$

where

$$(p\,a_\perp) = 0, \qquad (p\,b_\perp) = 0, \qquad (p\,p) = 0\,.$$

The gauge functions are superposition of the functions

$$f^{(1)} = (y^5)^4 \, e^{i(py)} + \text{h. c.}, \qquad (9)$$

$$f^{(2)} = e^{i(py)} + \text{h. c.}, \qquad (10)$$

$$(p\,p) = 0\,.$$

The gauge transformation (9) is a special case of (7), and (10) and (8) are identical. If now we make the step to the conformal space by taking the limit $y^5 \to 0$ we are left with the plane wave solution

$$A_j = b_{j\perp} e^{i(py)} + \text{h. c.}, \qquad A_5 = 0$$

of the electromagnetic field. The transformation generated by (10) is the conventional gauge transformation for the electromagnetic field (satisfying the LORENTZ condition).

We may summarize our results as follows. The unitary representations of the group SO(4,2) which belong the tensors of rank 1, describe a vector meson and a scalar field with arbitrary mass $m \neq 0$. A certain class of these representations admit gauge transformations, and therefore describe (in the free field case) only vector mesons. Moreover only this class contains also unitary representations which belong to mass $m=0$, i. e. the electromagnetic field. The origin of the gauge transformations of the vector potentials has therefore to be explained by the special structure of this class of irreducible unitary representations. Primarily gauge transformations do not depend on the condition of vanishing mass.

Über die Verschiebung zwischen dem α-Phosphoreszenz- und Fluoreszenz-Spektrum des Trypaflavins in Plexiglas

A. KAWSKI, H. WARDZINSKI und B. GORCZYNSKI

Physikalisches Institut der Pädagogischen Hochschule, Gdańsk, Polen

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Kürzlich wurde über neue Beobachtungen der Phosphoreszenz von Trypaflavin in Polymethylmethakrylat mit 15% Methanol (PMAM) bei Zimmertemperatur berichtet [1]. Die Methylmethakrylat-Lösungen von Trypaflavin (MAM) wurden in einem verschlossenen Glasrohr polymerisiert. Nach dem Zerschlagen der Glasgefäße beobachtet man, daß die Phosphoreszenz schnell verschwindet. Es wurde festgestellt, daß das Maximum der α-Phosphoreszenzbande nicht mit dem Maximum der Flucreszenzbande zusammenfällt, sondern in Richtung längerer Wellen um ca. 100 Å verschoben ist, ähnlich wie das schon früher von POHOSKI [2, 3] bei Fluorescein in Borsäure beobachtet wurde.

In der vorliegenden Arbeit wird das Auftreten der Verschiebung der Fluoreszenz- und α-Phosphoreszenzbanden von PMAM-Trypaflavinluminophoren im Vakuum und bei verschiedenem Luft- bzw. Stickstoff-Druck untersucht.

Die Zubereitung der festen PMAM-Lösungen von Trypaflavin erfolgte in derselben Weise wie früher beschrieben [1]. Die MAM-Lösungen wurden in planparallele Glasgefäße eingegossen und im Hochvakuum entgast. Die Entgasung erfolgte durch ein zyklisches fünfmaliges: Gefrieren — Abpumpen — Anfeuchten. Dann wurden die Lösungen unter verschiedenem Luft- bzw. Stickstoffdruck abgeschmolzen. Das Begasen der Lösung mit Stickstoff hält sie frei von Sauerstoff. Die Polymerisation begann bei einer Temperatur von ca. 70 °C und lief dann von selbst bei 30 °C weiter. Um eventuellen photochemischen Reaktionen vorzubeugen, fand der Prozeß im Dunkeln statt. Die Messungen der Fluoreszenz- und Phosphoreszenzspektren wurden mit Hilfe

[1] A. KAWSKI, Z. Naturforschg. **20 a**, 1734 [1965].

[2] R. POHOSKI, Bull. Acad. Polon. Sci. Cl. III, **10**, 505 [1962].

[3] J. GRZYWACZ u. R. POHOSKI, Z. Naturforschg. **19 a**, 440 [1964].